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Can New Physics Hide inside the Proton?

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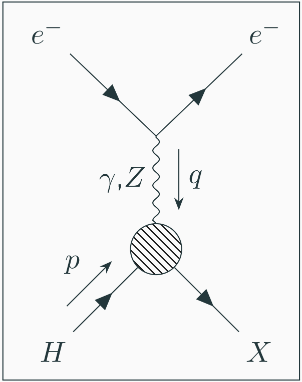
PDFs and their parameterization

Parton Distribution Functions

Our observable



$$\frac{d^2\sigma}{dQ^2 dy} \sim L^{\mu\nu} W_{\mu\nu}$$

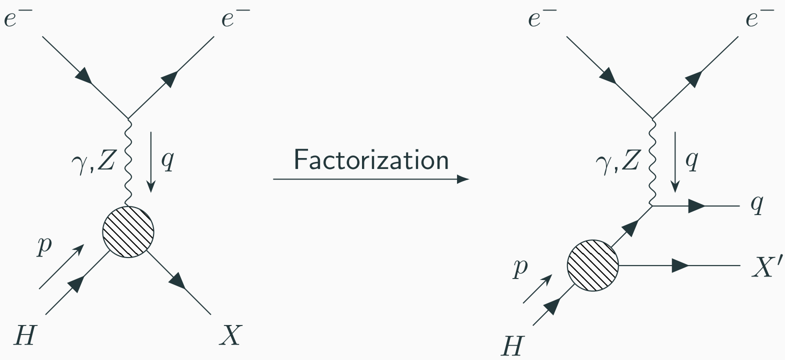
Where

$$W^{\mu\nu} = \frac{1}{4\pi} \int d^4y \sum_X e^{iq\cdot y} \cdot \langle H | j^\mu(y) | X \rangle \langle X | j^\nu(0) | H \rangle$$

Cannot be computed (entirely) in the framework of pQCD.

Useful kinematic variables:

$$\boxed{Q^2 = -q^2, \quad x = \frac{Q^2}{2p\cdot q}}$$

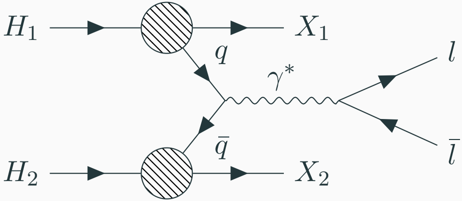


We can use the parton model to factorize the hard and soft physics (in the Bjorken limit of large Q):

$$\frac{d^2\sigma}{dQ^2dy} \sim \sum_q \hat{\sigma}^{\text{DIS}} \otimes f_q^{(H)}(x, Q^2) + \cdots \underbrace{\text{(Higher twists)}}_{\text{suppressed by } Q^2}$$

PDF Universality

Consider now the Drell-Yan process



PDFs depend on the initial state hadron, *not* the process.

$$\frac{d^2\sigma}{dQ^2dy} = \sum_{q_1,q_2} \hat{\sigma}^{\mathsf{DY}} \otimes f_{q_1}^{(H_1)} \otimes f_{q_2}^{(H_2)}$$

Fitting PDFs

PDFs are non-perturbative objects. They must be fitted to data in a global QCD analysis.

Take experimental measurements and compare to theory prediction.

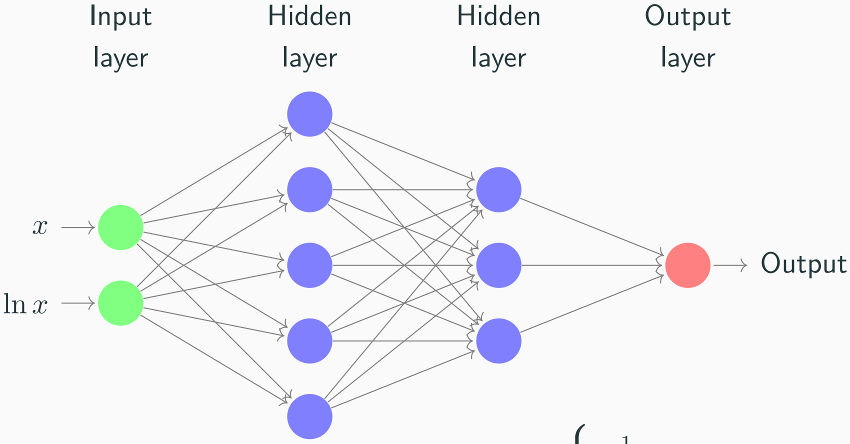
Find the PDFs that best fits the experimental data.

Fit only the x dependence at fixed scale $Q_0 = 1.65$GeV with the Q dependence determined by DGLAP:

$$Q^2 \frac{\partial f_i}{\partial Q^2}(x, Q^2) = \frac{\alpha_S(Q^2)}{2\pi} \int_x^1 \frac{d\xi}{\xi} P_{ij}\left(\frac{x}{\xi}, \alpha_S\right) f_j(\xi, Q^2)$$

which we use to evolve the PDF to the relevant kinematic scale.

NNPDF Parameterization



$$a_i^{(m)} = \sigma\left(\sum_{j=1}^{n} w_{ij} a_j^{(m-1)} + b\right)$$

$$\sigma(z) = \begin{cases} \frac{1}{1+e^{-z}} \\ \max(0, z) \end{cases}$$

No σ on output

Find the weights w_{ij} and biases b by genetic algorithm minimization of the χ^2

$$\chi^2 = (\text{data} - \text{theory})^T \text{cov}^{-1} (\text{data} - \text{theory})$$

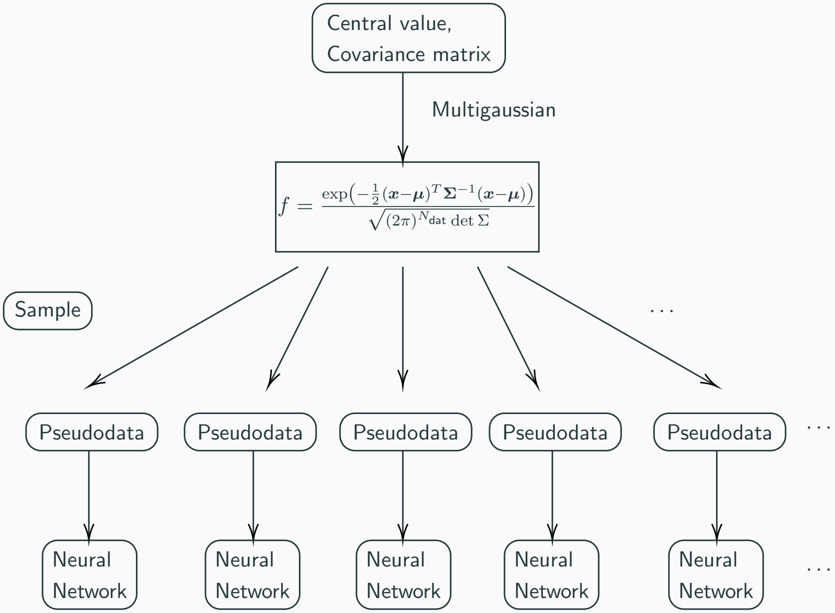
Initialize a bunch of w_j and b randomly and pick the best performing. Move the central value to the best performer and repeat.

Pseudodata replica method

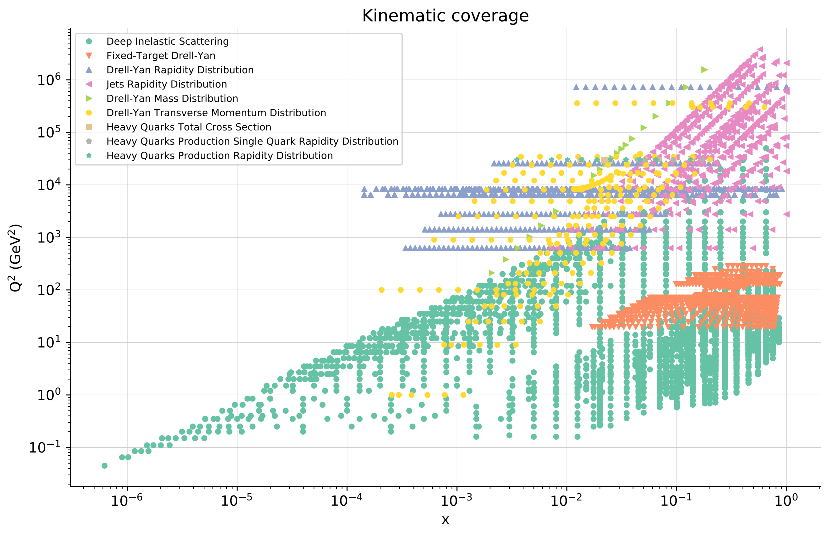
Fit PDFs to experimental data, which have errors. Need to propagate these errors down to the PDF fit level.

Do this by using *pseudodata replicas*. Data central value and covariance matrix define a multigaussian distribution.

Fit neural networks to samples of this distribution. Usually have $N_{\text{rep}} \sim 100$ *ensemble* of PDFs.

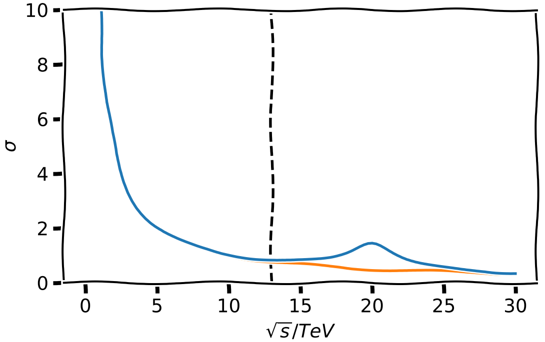

Central value,
Covariance matrix
Multigaussian
$f = \frac{\exp\left(-\frac{1}{2}(\boldsymbol{x}-\boldsymbol{\mu})^T \boldsymbol{\Sigma}^{-1}(\boldsymbol{x}-\boldsymbol{\mu})\right)}{\sqrt{(2\pi)^{N_{\mathrm{dat}}} \det \Sigma}}$
Sample
...
Pseudodata
Pseudodata
Pseudodata
Pseudodata
Pseudodata
...
Neural Network
Neural Network
Neural Network
Neural Network
Neural Network
...

Kinematic coverage of the data



Beyond Standard Model Effects

Possible BSM contamination from resonances beyond current collider kinematic reach.



Possible that parameterization adapts to fit away these BSM signals while trying to fit to a SM theory. Need to disentangle the PDF from possible contamination.

Standard Model Effective Field Theory

Standard Model Effective Field Theory

Treat the Standard Model as the low energy, IR limit of some UV complete theory

$$\mathcal{L} = \mathcal{L}_{\mathsf{SM}} + \sum_{d \geq 5} \sum_{i=1}^{N_d} \frac{a_i}{\Lambda^{d-4}} \mathcal{O}_i^{(d)}$$

Λ high energy cut-off (1 TeV), d the mass-dimension of operator $\mathcal{O}_i^{(d)}$. The $\{a_i\}$ called the Wilson Coefficients.

Ignore odd d values. Violate baryon/lepton number conservation. First non-trivial contribution at $d = 6$

Convenient because:

- Model independent. Uses same matter fields and gauge symmetry as the SM.
- For $d = 6$ and 3-flavours, minimal $\{\mathcal{O}_i^{(6)}\}$ basis fully determined (Warsaw basis[1]).
- Encompasses *any* UV complete theory that has SM as an IR limit.

[1] arXiv:1008.4884

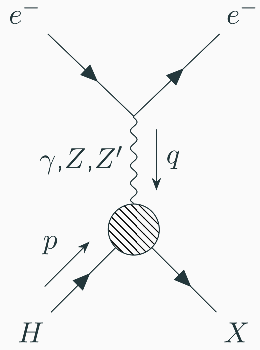
As a proof of concept consider only the subset of the Warsaw basis:

$$\mathcal{O}_{lu} = \left(\bar{l}_R \gamma^\mu l_R\right) \left(\bar{u}_R \gamma_\mu u_R\right) \quad , \quad \mathcal{O}_{ld} = \left(\bar{l}_R \gamma^\mu l_R\right) \left(\bar{d}_R \gamma_\mu d_R\right)$$
$$\mathcal{O}_{lc} = \left(\bar{l}_R \gamma^\mu l_R\right) \left(\bar{c}_R \gamma_\mu c_R\right) \quad , \quad \mathcal{O}_{ls} = \left(\bar{l}_R \gamma^\mu l_R\right) \left(\bar{s}_R \gamma_\mu s_R\right)$$

For l either an electron or muon.

Wish to find a confidence interval on the Wilson Coefficients for these operators using DIS data.

Introduces new Z' channel for DIS:



Which modifies the Neutral Current Structure functions:

$$F_2(x,Q^2) = F_2^{\rm SM}(x,Q^2) + \frac{x}{12e^4}\left[\left(4a_u e^2 \frac{Q^2}{\Lambda^2}(\overbrace{1}^{Z'/\gamma} + \overbrace{4K_Z s_W^4}^{Z'/Z}) + \overbrace{3a_u^2\frac{Q^4}{\Lambda^4}}^{Z'/Z'}\right)\left(u(x,Q^2)+\bar{u}(x,Q^2)\right) + \cdots\right]$$

$$F_3(x,Q^2) = F_3^{\rm SM}(x,Q^2) + \frac{1}{12e^4}\left[\left(4a_u e^2 \frac{Q^2}{\Lambda^2}(1 + 4K_Z s_W^4) + 3a_u^2\frac{Q^4}{\Lambda^4}\right)\left(u(x,Q^2)-\bar{u}(x,Q^2)\right) + \cdots\right]$$

Where

$$K_Z = \frac{Q^2}{4c_W^2 s_W^2 (Q^2 + M_Z^2)} \qquad c_W^2 = 1 - s_W^2 = \cos^2\theta_W.$$

Note $F_L = F_L^{\rm SM}$ since this analysis is leading order in the SMEFT.

Main analysis done up to $\mathcal{O}\left(\frac{1}{\Lambda^2}\right)$ because:

- Subleading compared to Z'/SM
- Will be corrected by $\mathcal{O}^{(8)}$ operators
- Results largely unchanged if you include them for a 1 dimensional analysis

Fixed PDF analysis

- Initially keep the PDF fixed as one fitted with SM theory. NNPDF31 nnlo as 0118 DISonly.
- Scan the SMEFT operator phase space by sampling (a_u, a_d, a_s, a_c).
- Then obtain a χ^2 value for how well modified DIS observable fits data.

Results look like:

(a_u, a_d, a_s, a_c)	χ^2
$(0, 0, 0, 0)$	3568.915
$(-0.18, 0, 0, 0)$	3571.693
$(0.9, 0.9, 0, 0)$	3583.612
$\vdots$	$\vdots$

Since structure functions are linear in Wilson Coefficient (a_i), the χ^2 is quadratic in a_i. Can represent the χ^2 as a quadratic form.

$$\chi^2(a;\beta) = \chi_0^2 + \frac{1}{2}(a - a_0)^T H (a - a_0)$$

H the Hessian, a_0 position of minimum and χ_0^2 value at minimum. Fit to χ^2 values using least squares to obtain fit parameters β $(\dim \beta = 15)$.

Find β by minimizing square difference.

$$\beta = \arg\min \sum_{i}^{N_{BP}} ||y_i - \chi^2(a_i;\beta)||^2$$

Can obtain analytic solution for β, since functional form is polynomial in fit parameters:

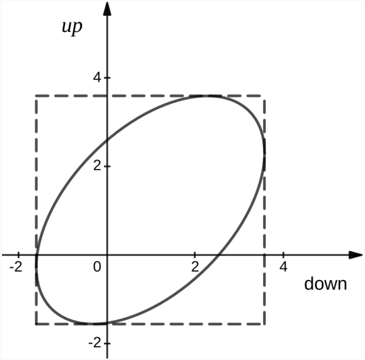
$$\beta = (X^T X)^{-1} X^T \vec{y}$$

for X the design matrix and $\vec{y}$ the vector of data to fit to.

To find constraints on the wilson coefficients, consider $\Delta\chi^2$. A region of 90% confidence is defined by

$$\Delta\chi^2 = \frac{1}{2}(a - a_0)^T H(a - a_0) = 7.779$$

Which defines a $\dim 3$ ellipsoid embedded in $\mathbb{R}^4$. The extremities along principal axes give our bounds.

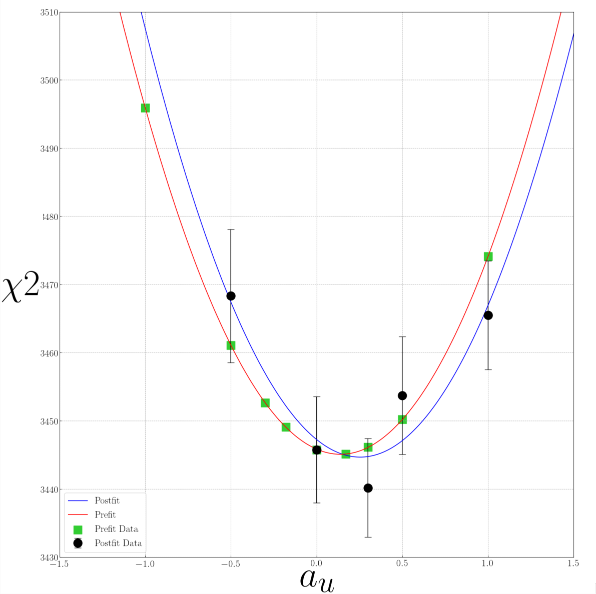


Fitting PDFs with SMEFT theory

Now allow the PDF to change in the presence of SMEFT operators. Requires employing NNPDF methodology to fit PDFs from scratch.

- Modify theory prediction ($d^2\sigma$) with BSM operators (generated by APFEL)
- Use NNPDF methodology to obtain PDF fits using the modified theory.

Results



Choose points along each of the principle directions e.g $(a_u, 0, 0, 0)$.

Four Dimensional Analysis (SM PDFs)

Perform the analysis in the presence of all 4 SMEFT operators (marginalized bounds) and compare with individual bounds.

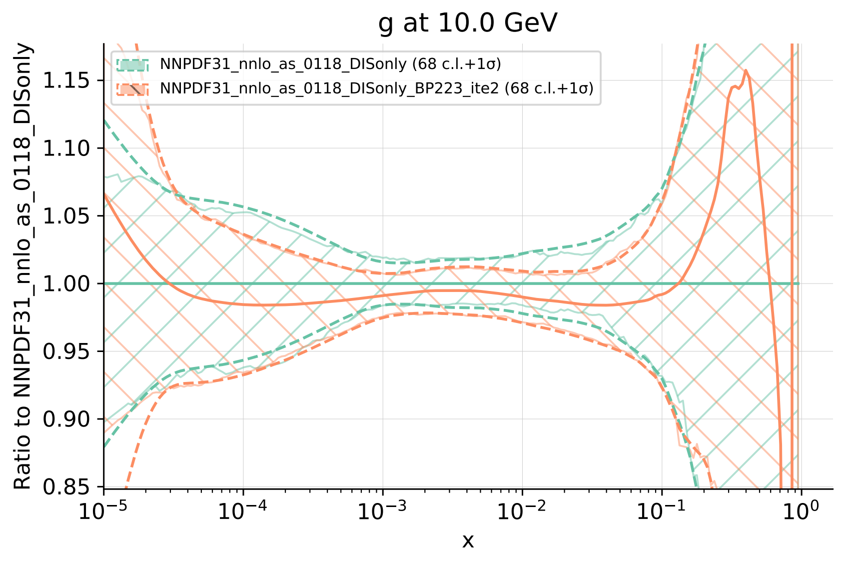
Flavour	Individual Bounds	Marginalized Bounds
up	$[-0.1, +0.4]$	$[-2.3, +1.4]$
down	$[-1.6, +0.4]$	$[-13, +3.9]$
strange	$[-2.8, +4.2]$	$[-18, +29]$
charm	$[-2.6, +1.2]$	$[-13, +7.0]$

Four Dimensional Analysis (SMEFT PDFs)

Bounds obtained with PDFs fitted with SMEFT operators

Flavour	Individual Bounds	Marginalized Bounds
up	$[0.0, +0.5]$	$[-0.4, +2.4]$
down	$[-1.1, +0.8]$	$[-4.4, +4.5]$
strange	$[-4.5, +3.6]$	$[-61, +39]$
charm	$[-2.4, +0.7]$	$[-29, +2.7]$

Gluon fitted at $(-1.3, 1.3, 0, 0)$



Next-Steps

Structure functions at NLO

NLO SMEFT corrections for structure functions. Need to include diagrams for the hard process:

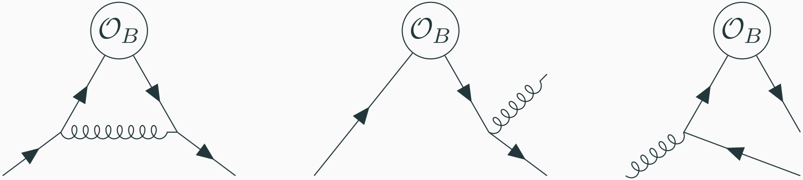
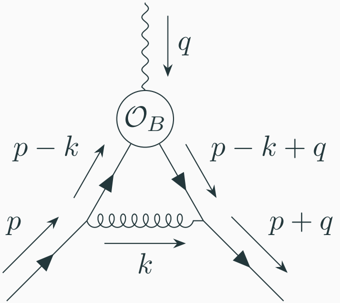
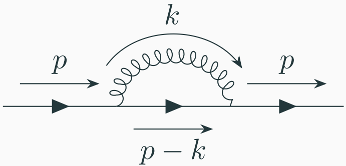


Figure 1: Virtual emission, real emission, and gluon boson fusion

Where UV divergence cancel the mass divergences (KLN theorem), while collinear divergences result in running of the PDF after collinear subtraction.

Non-renormalization of Wilson Coefficients with α_S





Adding counter-terms in dimensional regularization we have (after field redefinition)

$$\mathcal{L} \mapsto \widetilde{\mathcal{L}} = \bar{q}_R i\partial\!\!\!/ q_R + \frac{1+\delta a}{1+\delta\psi}\frac{a}{4\Lambda^2}\bar{q}_R\gamma^\mu(1+\lambda\gamma^5)q_R\bar{l}\gamma_\mu(1+\lambda'\gamma^5)l$$

with

$$\boxed{\delta a = \delta\psi = \frac{\alpha_S C_F}{4\pi\epsilon}}$$

Including high mass DY measurements

Experiment	$\sqrt{s}$	Range	Lumi (fb^{-1})	Reference
ATLAS	7	$m_{ee} < 1.5$TeV	4.9	1305.4192
	8	$m_{ll} < 1.5$TeV	20.3	1606.01736
CMS	7	$m_{ee}, m_{\mu\mu} < 1.5$TeV	4.5(e)+ 4.8(μ)	1310.7291
	8	$m_{ll} < 2$TeV	19.7	1412.1115
	13	$m_{ee}, m_{\mu\mu} < 3$TeV	2.8(e)+ 2.3(μ)	1812.10529

With theory generation handled by MadGraph 5 + SMEFTsim.
But only LO in the SMEFT for pp processes.

Updated methodology

Wish to simultaenously constrain PDF with the Wilson coefficients. Treat these two objects on an equal footing.

As we saw with SMEFT corrections, PDF modification is not very severe. Say the PDF fitted alongside the Wilson coefficients is a linear combination of the baseline.

Can do this by reweighting a baseline PDF set. Take as a baseline, NNPDF31 nnlo as 0118 DISonly $\{\bar{f}^{(k)} : k = 1, \ldots, N_{\text{rep}}\}$.

$$\boxed{f(x, Q^2) = \sum_{k=1}^{N_{\text{rep}}} w_k \bar{f}^{(k)}(x, Q^2)}$$

$$\underbrace{\sum_{k=1}^{N_{\text{rep}}} w_k = 1}_{\text{QCD sum rules}} \qquad \underbrace{w_k \geq 0 \; \forall k}_{\text{positivity of observables}}$$

Finding the weights

Our theory prediction now looks like:

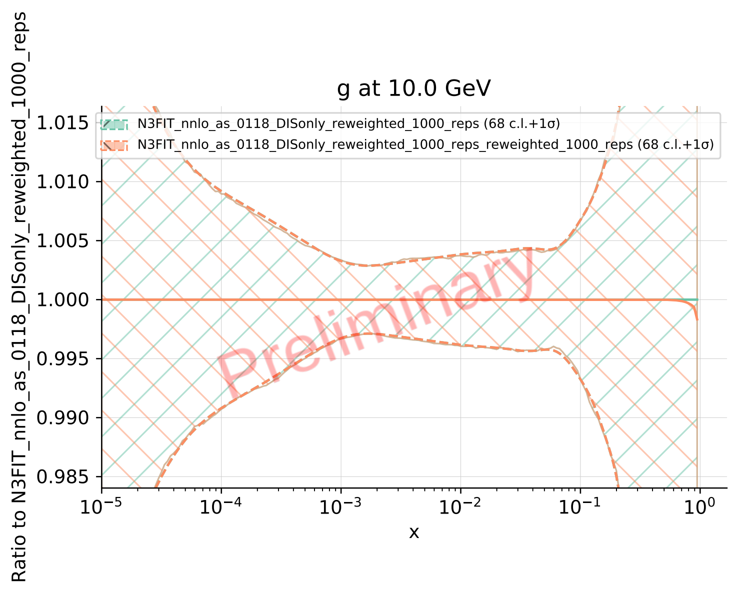
$$T = \hat{\sigma}_{\text{SM}} \otimes \bar{f}^{(0)} + \sum_{k=1}^{N_{\text{rep}}} w_k \, \hat{\sigma}_{\text{SM}} \otimes \left(\bar{f}^{(k)} - \bar{f}^{(0)}\right) + \sum_{n=1}^{N_{\text{op}}} \frac{c_n}{\Lambda^2} \hat{\sigma}_n \otimes \bar{f}^{(0)}.$$

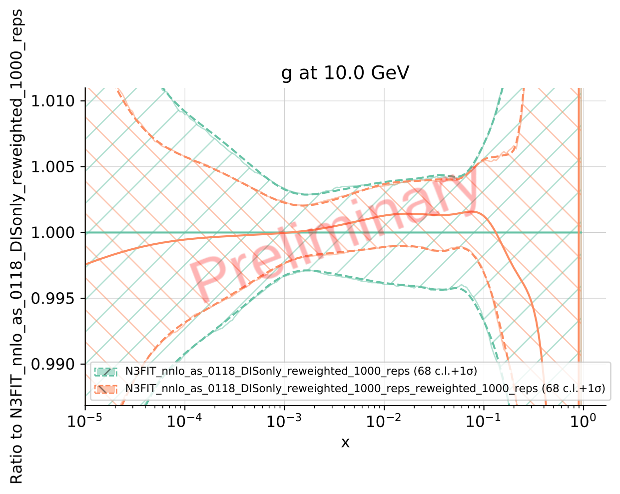
Which modifies the χ^2 to be:

$$\chi^2 = \chi^2(\{w_k\}, \{c_n\})$$

Find the best fit w_k and c_n by minimizing the (convex) χ^2.

Generate a *reweighted ensemble* by fitting to pseudodata replicas.


g at 10.0 GeV
N3FIT_nnlo_as_0118_DISonly_reweighted_1000_reps (68 c.l.+1σ)
N3FIT_nnlo_as_0118_DISonly_reweighted_1000_reps_reweighted_1000_reps (68 c.l.+1σ)
Ratio to N3FIT_nnlo_as_0118_DISonly_reweighted_1000_reps
1.015
1.010
1.005
1.000
0.995
0.990
0.985
10⁻⁵
10⁻⁴
10⁻³
10⁻²
10⁻¹
10⁰
x
Preliminary



Flavour	Bounds
up	$[-0.92, 1.30]$
down	$[-6.72, 3.38]$
strange	$[-18.9, 6.02]$
charm	$[-13.22, -1.35]$

Conclusions

- Demonstrated of a proof of concept. We can systematically disentangle PDFs from BSM contamination.
- Present first bounds on Wilson coefficients obtained in this way.
- Outlined new methodology to constrain PDFs and Wilson coefficients on an equal footing, paving the way for a first truly global fit.

χ^2 vs q_{max} for $(-1.3, 1.3, 0, 0)$